

$(g - 2)_\mu$ and Supersymmetry

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Tucson, 03/2004

based on collaboration with
D. Stöckinger and G. Weiglein

1. Introduction
2. The anomalous magnetic moment of the Muon
3. Calculation of MSSM two-loop corrections
4. MSSM two-loop results (incl. very recent results)
5. Conclusions

1. Introduction

Q: Which Lagrangian describes the world?

Q': How can one distinguish SM and MSSM?

A: Two possible ways:

- Search for new SUSY particles

new SUSY particles found
↔
SM ruled out

Problem:

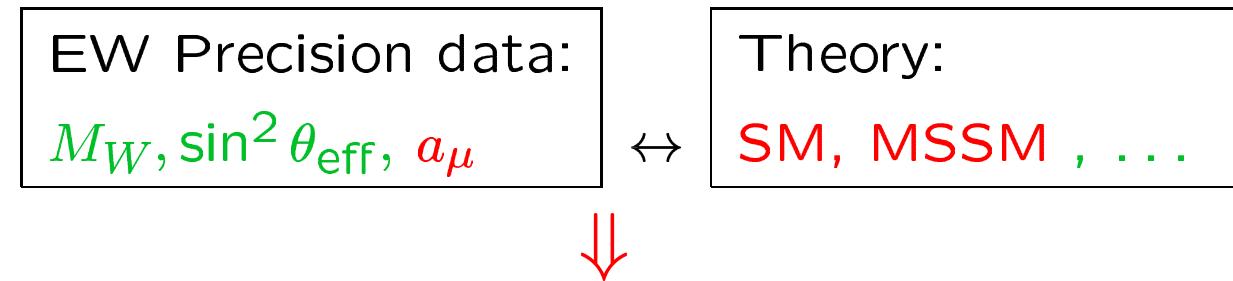
SUSY particles are too heavy for todays colliders, only upper limits of $\mathcal{O}(100 \text{ GeV})$.

- waiting for Tevatron (2005...?)
- waiting for LHC (2007?)

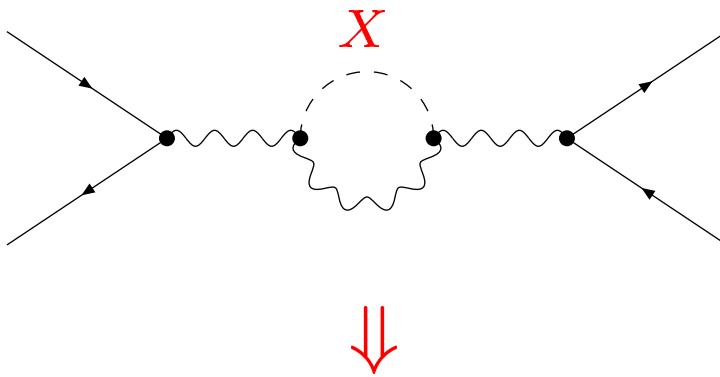
- Search for indirect effects of SUSY via Precision Observables

Precision Observables (POs):

Comparison of electro-weak precision observables with theory:



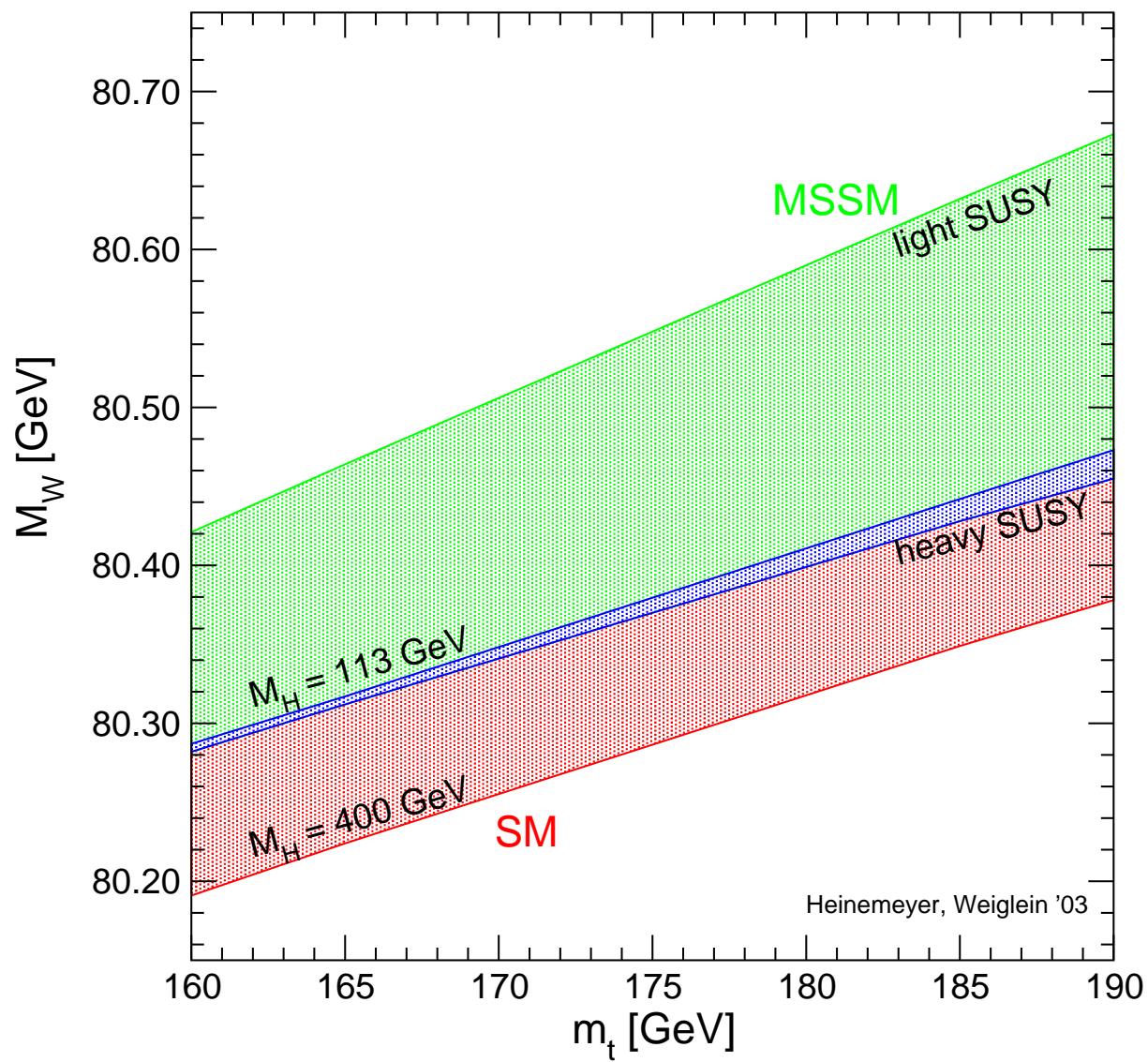
Test of theory at quantum level: **Sensitivity to loop corrections**



Very high accuracy of measurements and theoretical predictions needed

- Which model fits better?
- Does the prediction of a model contradict the experimental data?

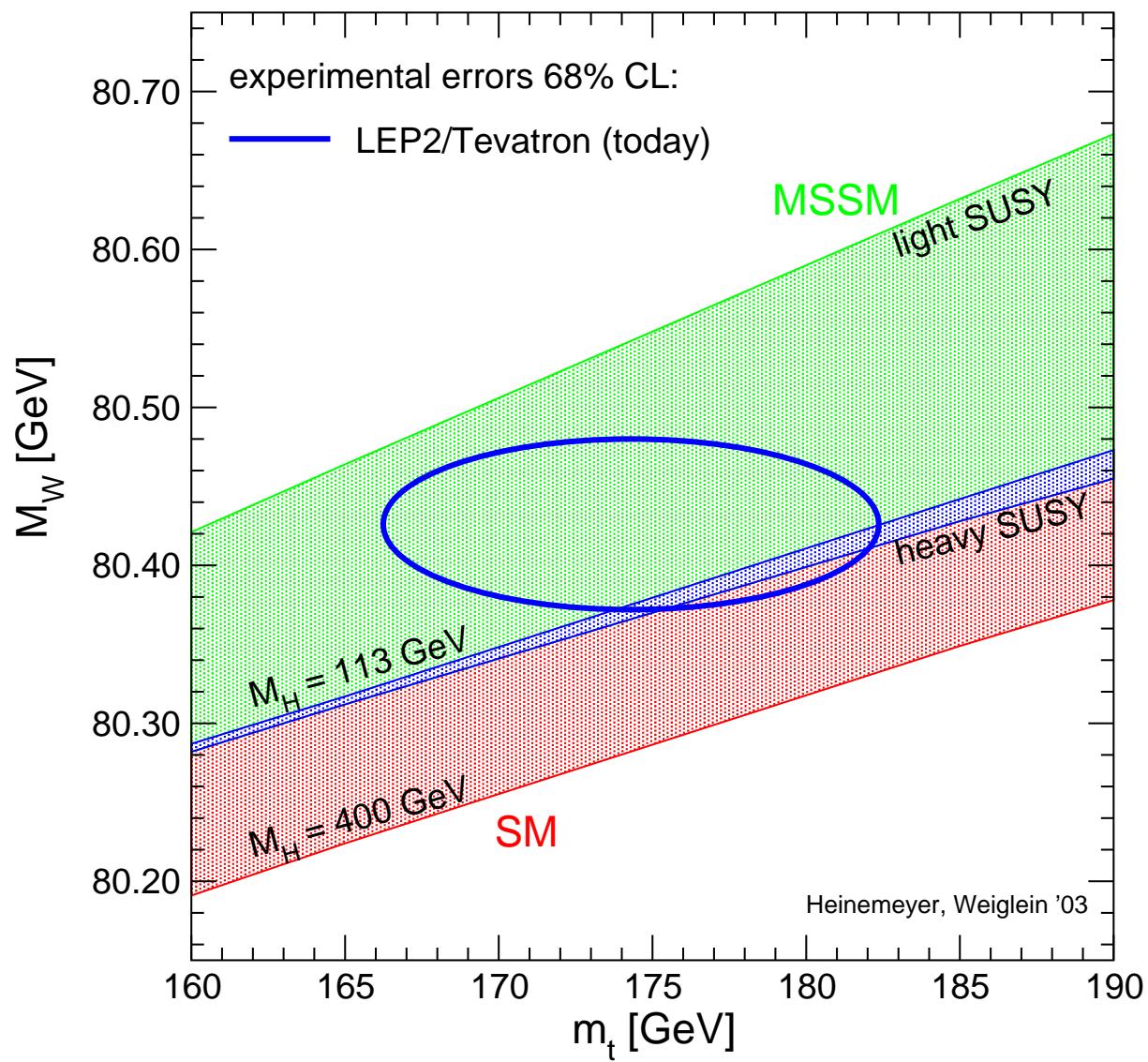
Example: Prediction for M_W in the SM and the MSSM :



MSSM uncertainty:
unknown masses
of SUSY particles

SM uncertainty:
unknown Higgs mass

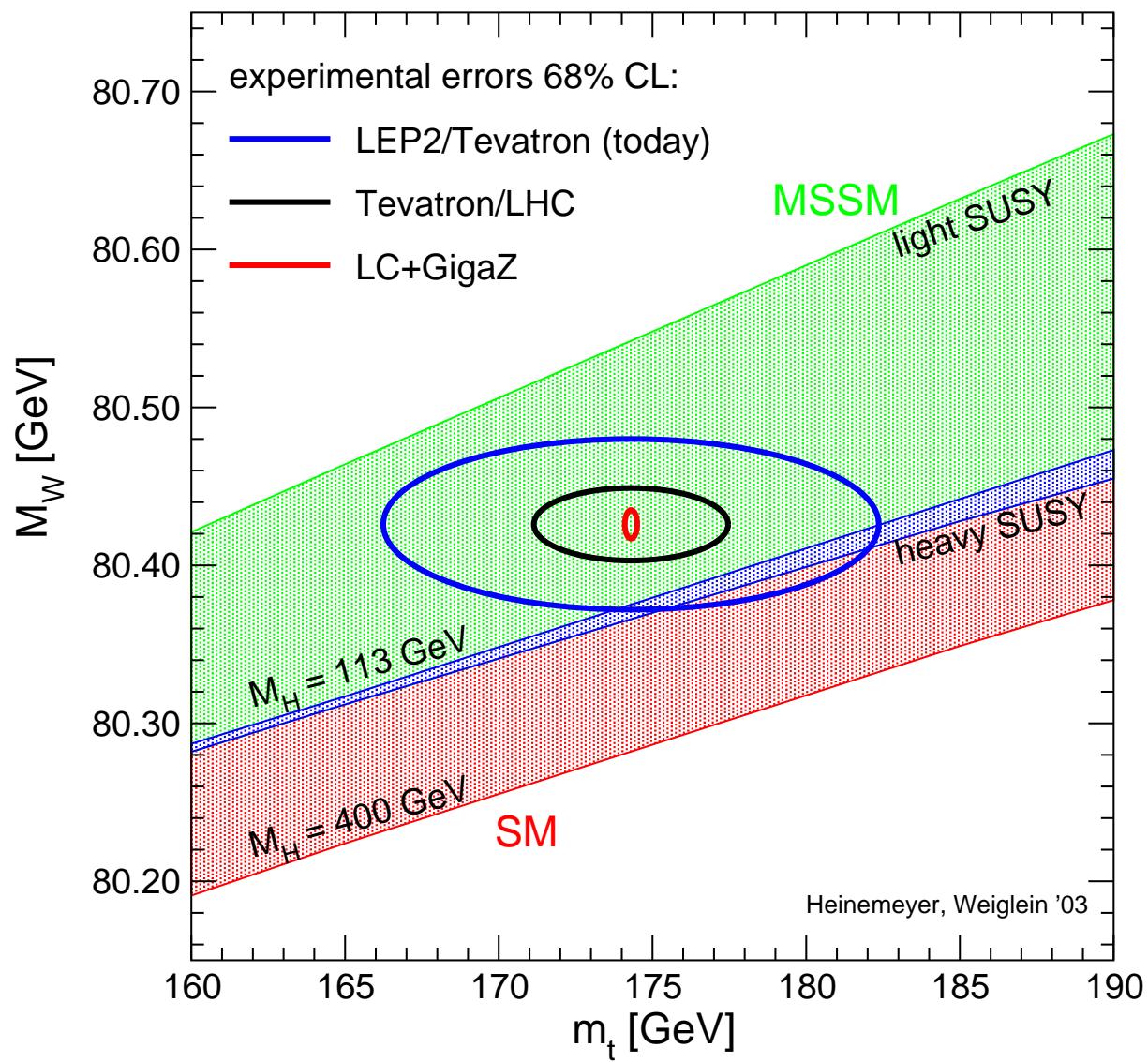
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The Minimal Supersymmetric Standard Model (MSSM)

Superpartners for Standard Model particles

$[u, d, c, s, t, b]_{L,R}$	$[e, \mu, \tau]_{L,R}$	$[\nu_{e,\mu,\tau}]_L$	Spin $\frac{1}{2}$
$[\tilde{u}, \tilde{d}, \tilde{c}, \tilde{s}, \tilde{t}, \tilde{b}]_{L,R}$	$[\tilde{e}, \tilde{\mu}, \tilde{\tau}]_{L,R}$	$[\tilde{\nu}_{e,\mu,\tau}]_L$	Spin 0
g	$\underbrace{W^\pm, \textcolor{red}{H}^\pm}_{\textcolor{blue}{W^\pm, H^\pm}}$	$\underbrace{\gamma, Z, \textcolor{red}{H}_1^0, H_2^0}_{\textcolor{black}{\gamma, Z, H_1^0, H_2^0}}$	Spin 1 / Spin 0
\tilde{g}	$\tilde{\chi}_{1,2}^\pm$	$\tilde{\chi}_{1,2,3,4}^0$	Spin $\frac{1}{2}$

Enlarged Higgs sector: Two Higgs doublets

Problem in the MSSM: many scales

Enlarged Higgs sector: Two Higgs doublets

$$H_1 = \begin{pmatrix} H_1^1 \\ H_1^2 \end{pmatrix} = \begin{pmatrix} v_1 + (\phi_1 + i\chi_1)/\sqrt{2} \\ \phi_1^- \end{pmatrix}$$

$$H_2 = \begin{pmatrix} H_2^1 \\ H_2^2 \end{pmatrix} = \begin{pmatrix} \phi_2^+ \\ v_2 + (\phi_2 + i\chi_2)/\sqrt{2} \end{pmatrix}$$

$$V = m_1^2 H_1 \bar{H}_1 + m_2^2 H_2 \bar{H}_2 - m_{12}^2 (\epsilon_{ab} H_1^a H_2^b + \text{h.c.})$$

$$+ \underbrace{\frac{g'^2 + g^2}{8}}_{\text{gauge couplings, in contrast to SM}} (H_1 \bar{H}_1 - H_2 \bar{H}_2)^2 + \underbrace{\frac{g^2}{2}}_{\text{gauge couplings, in contrast to SM}} |H_1 \bar{H}_2|^2$$

physical states: h^0, H^0, A^0, H^\pm

Goldstone bosons: G^0, G^\pm

Input parameters:

$$\tan \beta = \frac{v_2}{v_1}, \quad M_A^2 = -m_{12}^2(\tan \beta + \cot \beta)$$

→ Prediction for m_h :

MSSM tree-level bound: $m_h < M_Z$, excluded by LEP Higgs searches

Large radiative corrections:

Dominant one-loop corrections: $\sim G_\mu m_t^4 \ln\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}\right)$

Measurement of m_h , Higgs couplings ⇒ test of the theory

LHC: $\Delta m_h \approx 0.2$ GeV, LC: $\Delta m_h \approx 0.05$ GeV

⇒ m_h will be (the best?) electroweak precision observable

Present status of m_h prediction in the MSSM:

Feynman-diagrammatic result:

complete one-loop, leading + subleading two-loop

FeynHiggs2.1 (www.feynhiggs.de)

[S. H., W. Hollik, G. Weiglein '98, '00, '02]

[T. Hahn, S. H., W. Hollik, G. Weiglein '04]

⇒ used for later Higgs evaluation

Upper bound on m_h in the rMSSM:

“Unconstrained (real) MSSM”:

M_A , $\tan\beta$, 5 parameters in \tilde{t} - \tilde{b} sector, μ , $m_{\tilde{g}}$, M_2

FeynHiggs $\Rightarrow m_h \lesssim 133$ GeV

[S. H., W. Hollik, G. Weiglein '99] ,

[G. Degrassi, S. H., W. Hollik, P. Slavich, G. Weiglein '02]

for $m_t = 175$ GeV, no theoretical uncertainties included

Remaining theoretical uncertainties in prediction for m_h :

[G. Degrassi, S. H., W. Hollik, P. Slavich, G. Weiglein '02] ,

[M. Frank, S. H., W. Hollik, G. Weiglein '02]

– From unknown higher-order corrections:

$$\Rightarrow \Delta m_h \approx 3 \text{ GeV}$$

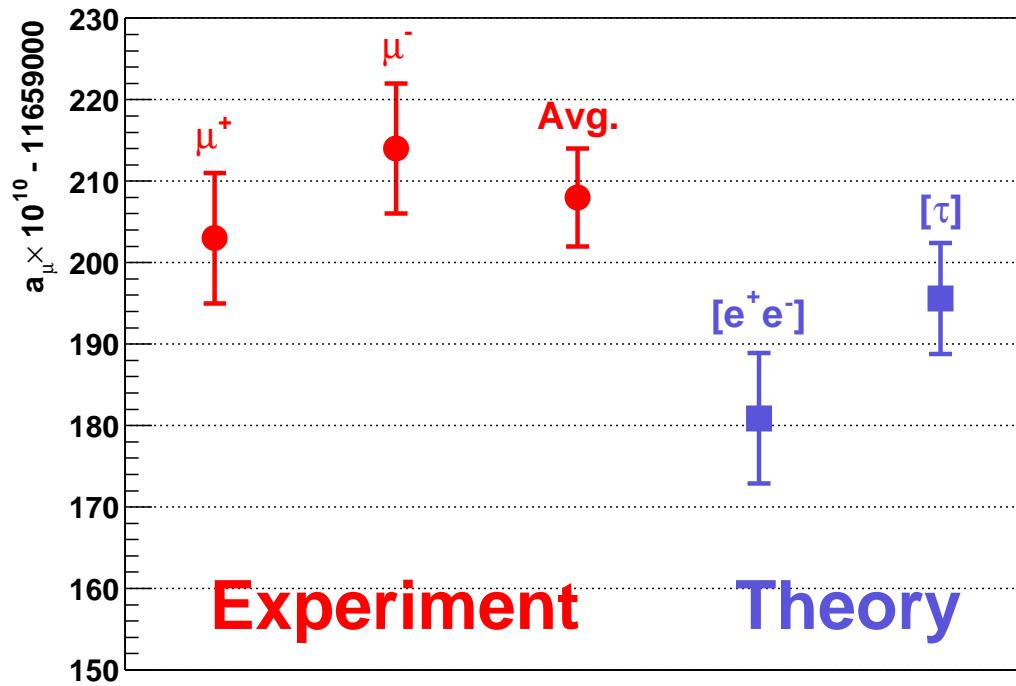
– From uncertainties in input parameters

$$m_t, \dots, M_A, \tan\beta, m_{\tilde{t}_1}, m_{\tilde{t}_2}, \theta_{\tilde{t}}, m_{\tilde{g}}, \dots$$

$$\Delta m_t \approx 5 \text{ GeV} \Rightarrow \Delta m_h \approx 5 \text{ GeV}$$

2. The anomalous magnetic moment of the muon: $a_\mu = (g - 2)_\mu / 2$

Overview about the experimental and SM (theory) result:
[*g-2 Collaboration, hep-ex/0401008*]

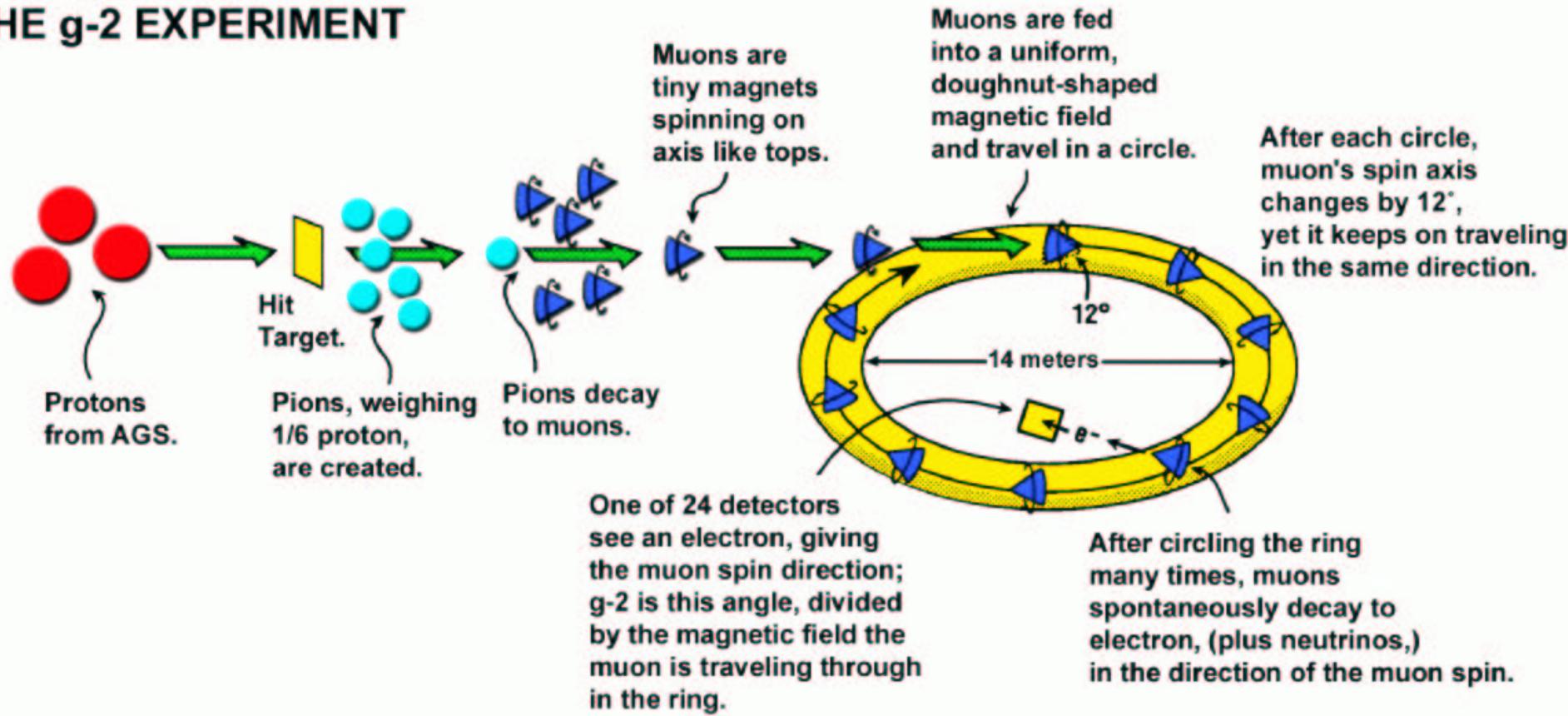


$$\text{Experiment : } a_\mu^{\text{exp}} = (11\,659\,208 \pm 6) \times 10^{-10}$$

$$\text{SM Theory : } a_\mu^{\text{theo}} \approx (11\,659\,181 \pm 8) \times 10^{-10}$$

The $(g - 2)_\mu$ experiment:

LIFE OF A MUON: THE g-2 EXPERIMENT



Coupling of muon to magnetic field : $\mu - \mu - \gamma$ coupling

$$\bar{u}(p') \left[\gamma^\mu F_1(q^2) + \frac{i}{2m_\mu} \sigma^{\mu\nu} q_\nu F_2(q^2) \right] u(p) A_\mu \quad F_2(0) = (g - 2)_\mu$$

The SM theory evaluation:

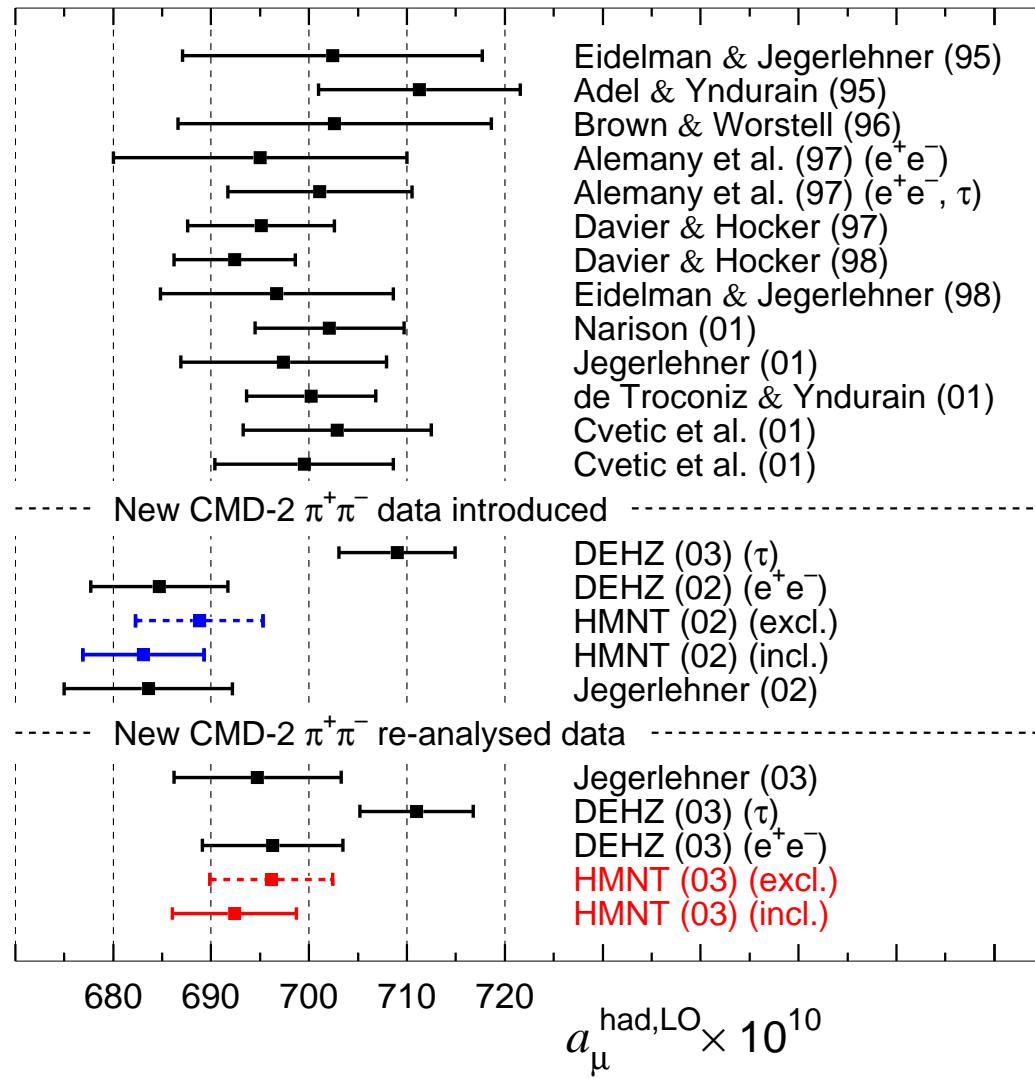
Source	contr. to $a_\mu[10^{-10}]$		
LO hadr.	696.3 ± 7.2	(e^+e^-)	[Davier, Eidelman, Höcker, Zhang '03]
	711.0 ± 5.8	(τ)	[Davier, Eidelman, Höcker, Zhang '03]
	694.8 ± 8.6	(e^+e^-)	[Ghozzi, Jegerlehner '03]
	691.7 ± 6.1	(e^+e^-)	[Hagiwara, Martin, Nomura, Teubner '03]
LBL	8 ± 4		[Knecht, Nyffeler '02]
	13.6 ± 2.5	tbc	[Melnikov, Vainshtein '03]
EW 1L	19		
EW 2L	-4		[Czarnecki, Krause, Marciano '98]
exp. res.	6		

difference of τ based and e^+e^- based LO hadr. is about $2 - 3\sigma$

“Isospin breaking effects” in τ based evaluation of LO hadr. not properly under control ? [Ghozzi, Jegerlehner '03]

⇒ concentrate on e^+e^- data

The $a_\mu^{\text{had,LO}}$ history:



Deviation of a_μ^{exp} and a_μ^{theo} (e^+e^-):

$a_\mu^{\text{exp}} - a_\mu^{\text{theo}}$ [10^{-10}]	[σ]	
27 ± 10.0	2.7	[Davier, Eidelman, Höcker, Zhang '03]
28 ± 11.1	2.5	[Ghozzi, Jegerlehner '03]
32 ± 9.5	3.4	[Hagiwara, Martin, Nomura, Teubner '03]
26 ± 8.9	3.0	including [Melnikov, Vainshtein '03]

SUSY corrections at 1L:

→ T

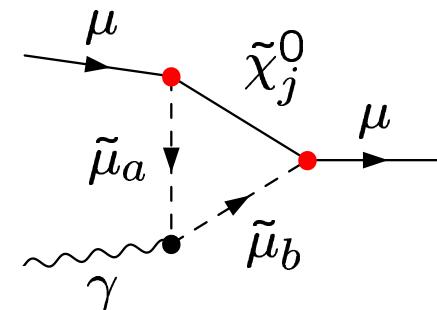
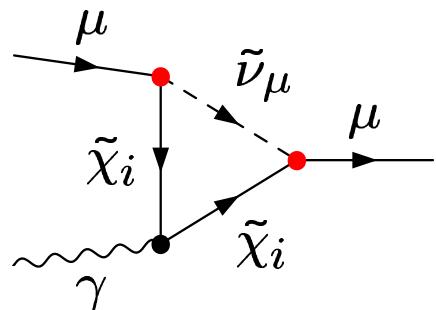
$$a_\mu^{\text{SUSY}} \approx 13 \times 10^{-10} \left(\frac{100 \text{ GeV}}{\tilde{M}} \right)^2 \tan \beta \text{ sign}(\mu)$$

\tilde{M} : generic SUSY mass scale

SUSY corrections at 2L: ??

(→ our calculation)

Feynman diagrams for MSSM 1L corrections:



- Diagrams with chargino/sneutrino exchange
- Diagrams with neutralino/smuon exchange

Enhancement factor as compared to SM:

$$\mu - \tilde{\chi}_i^\pm - \tilde{\nu}_\mu : \sim m_\mu \tan \beta$$

$$\mu - \tilde{\chi}_j^0 - \tilde{\mu}_a : \sim m_\mu \tan \beta$$

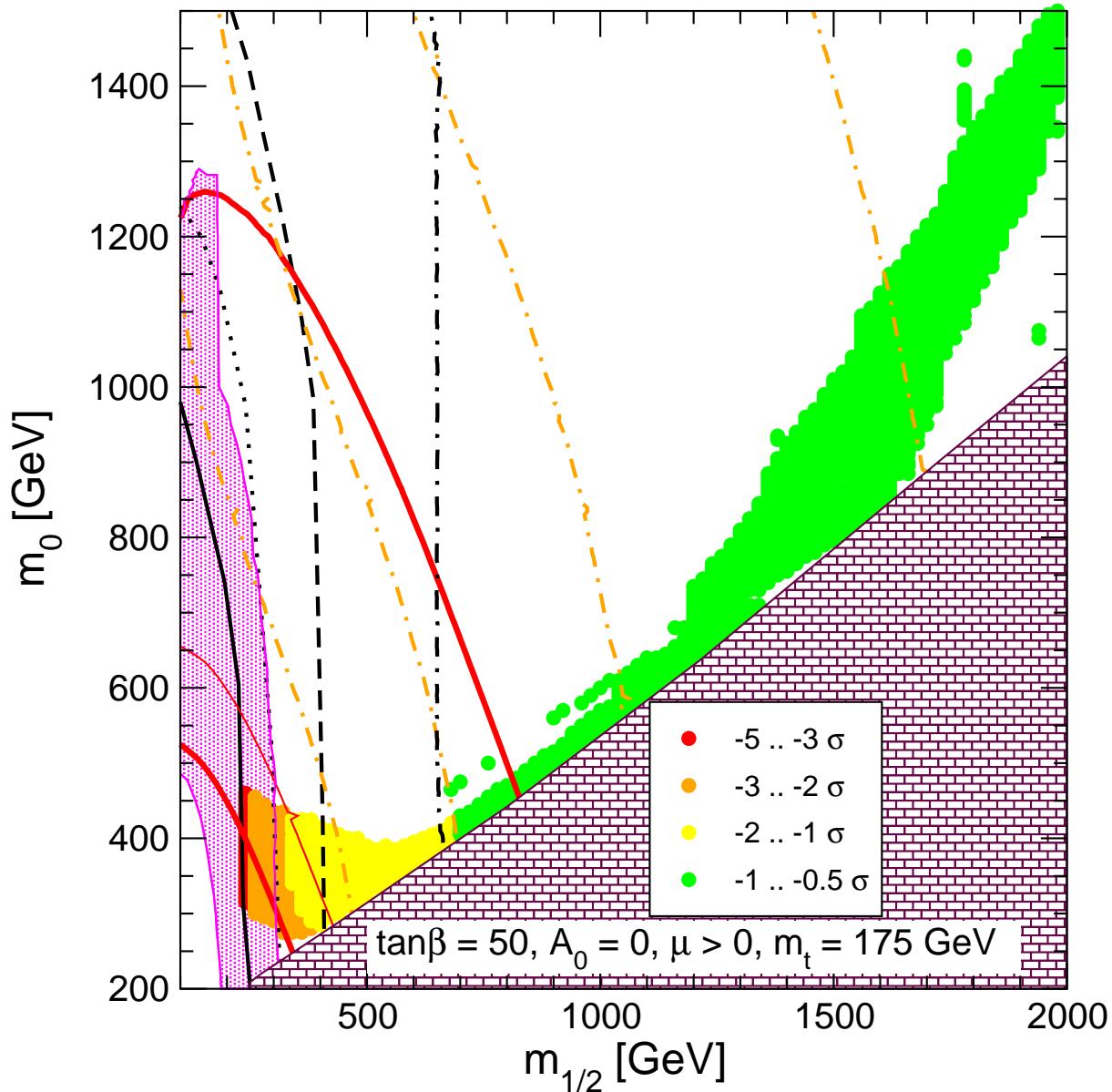
$$\text{SM, EW 1L: } \frac{\alpha}{\pi} \frac{m_\mu^2}{M_W^2}$$

$$\text{MSSM, 1L: } \frac{\alpha}{\pi} \frac{m_\mu^2}{\tilde{M}^2} \times \tan \beta$$

SUSY could easily explain the “discrepancy”
 a_μ can provide bounds on SUSY parameters

$\rightarrow T$

Example for bounds on SUSY parameters: mSUGRA



mSUGRA:

$\tan\beta = 50$, $A_0 = 0$, $\mu > 0$

$\text{BR}(h \rightarrow WW^*)$, MSSM/SM

[J.Ellis, S.H., K.Olive, G.Weiglein
'02]

e^+e^- : $\delta a_\mu = (33.9 \pm 11.2)$

τ : $\delta a_\mu = (16.7 \pm 10.7)$

Already known:

QED corrections to $a_{\mu}^{\text{SUSY}}(1\text{L})$: $\sim -7\%$

[*G. Degrassi, G. Giudice '98*]

Already known:

Approximation of leading terms can be very large, up to $\sim 20 \times 10^{-10}$

[*C. Chen, C. Geng '01*] , [*A. Arhrib, S. Baek '01*]

However: results disagree by a factor of 4!

Questions for numerical evaluation:

Q1: How large are the complete contributions?

Q2: What happens if experimental constraints are taken into account?

Our goal:

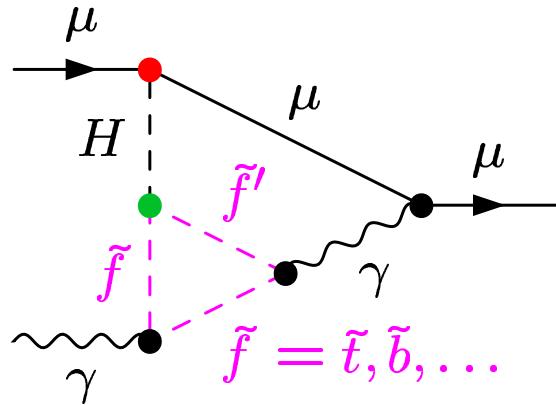
all corrections to SM/THDM diagrams with a
closed fermion or scalar fermion loop

→ T

Possible enhancement by:

- t, b, τ Yukawa couplings
- large μ and/or A_f and/or $\tan\beta$ (in couplings)
- small $m_{\tilde{t}}, m_{\tilde{b}}, m_{\tilde{\tau}}$

Example: Barr-Zee type diagram:



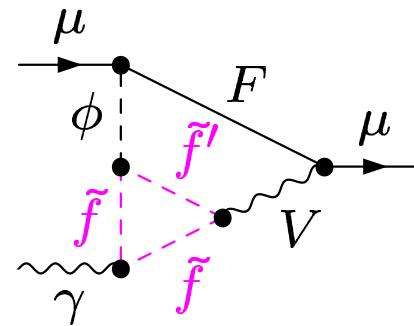
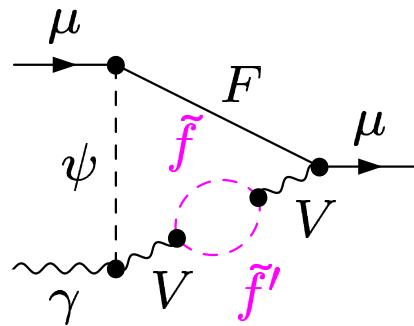
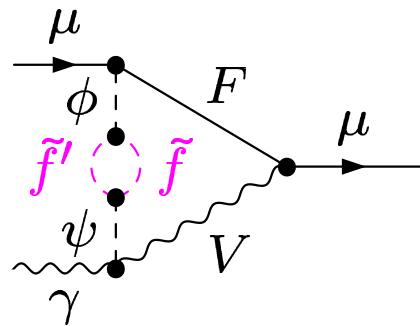
$$\begin{aligned}\rightarrow & m_\mu \tan\beta \\ \rightarrow & \mu m_t, m_b A_b \tan\beta\end{aligned}$$

⇒ Enhancement:

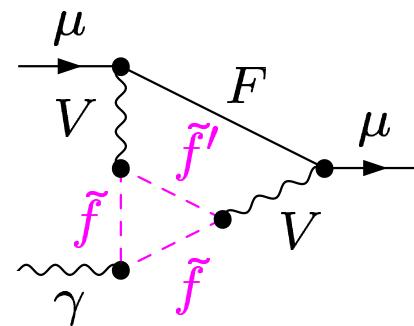
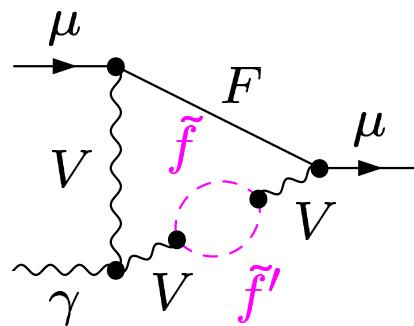
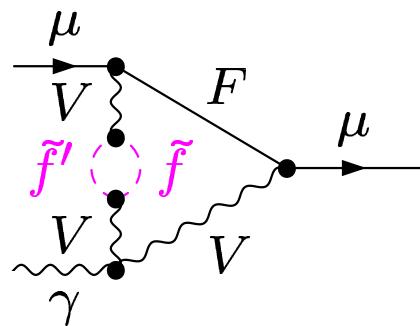
$$a_\mu \sim \tan\beta \frac{\mu m_t}{M_H m_{\tilde{t}}}$$

$$a_\mu \sim \tan^2\beta \frac{m_b A_b}{M_H m_{\tilde{b}}}$$

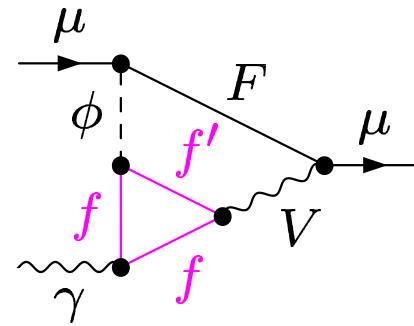
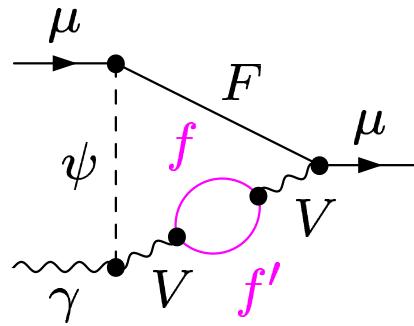
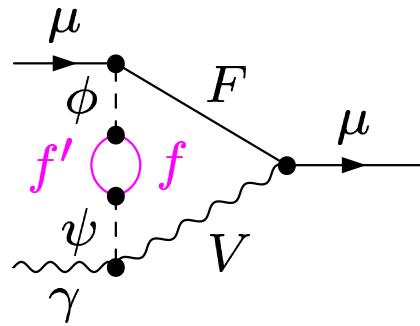
⇒ can be large even if SUSY 1L is small ! (other particles in the loops)



type: $(\tilde{f}V\phi)$



type: $(\tilde{f}VV)$



type: $(fV\phi)$

3. Calculation of MSSM two-loop corrections

Overview:

1. Generate Feynman diagrams for $\mu\mu\gamma$ at two-loop
2. Extract contribution of $(g - 2)_\mu$ (given in terms of two-loop integrals)
3. Expand integrals in $m_\mu \ll M_{\text{weak}}, m_{\tilde{f}}$
4. \Rightarrow analytic expression in $M_{\text{weak}}, M_\phi, m_{\tilde{f}}$

1. Generate Feynman diagrams for $\mu\mu\gamma$ at two-loop

- use *FeynArts* (www.feynarts.de)

[*J. Küblbeck, M. Böhm, A. Denner '90*]

[*T. Hahn '00 - '03*]

- use *MSSM* model file

[*T. Hahn, C. Schappacher '01*]

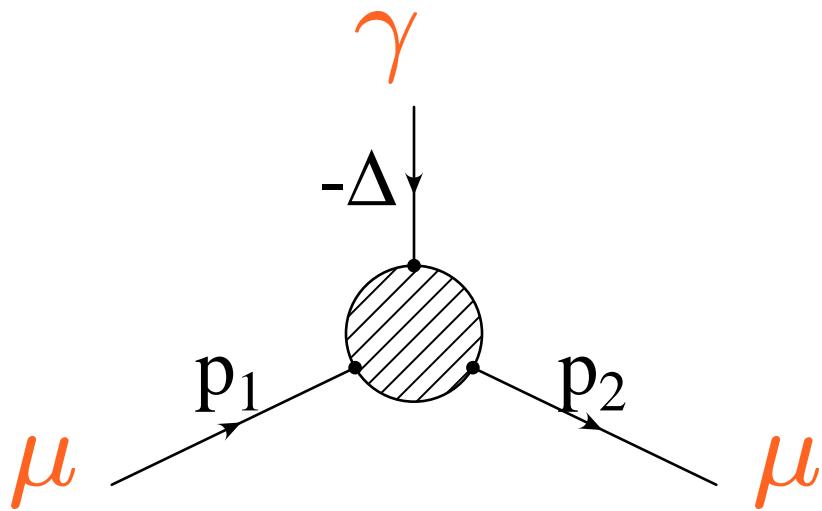
⇒ obtain all *two-loop* diagrams

one-loop diagrams with counter term insertion

one-loop diagrams for renormalization

⇒ transform *diagrams to amplitudes*

2. Extract contribution of $(g - 2)_\mu$



Amplitude: $M_\mu(p, \Delta)$

Expansion in γ momentum:

$$M_\mu(p, \Delta) \approx V_\mu(p) + \Delta^\nu T_{\nu\mu}(p)$$

($V_\mu, T_{\nu\mu}$ given in terms
of 2L self-energies)

next step: project out $(g - 2)_\mu$:

$$a \sim \frac{1}{m_\mu^2} \text{Tr} [P^\mu V_\mu + Q^\nu \mu T_{\nu\mu}]$$

Next step:

- Dirac algebra
- traces, ...
- reduction to a “basic” set of 2L integrals

⇒ $a = (g - 2)/2$ is given in terms of 2L integrals:

$$a = (g - 2)/2 = C_1 \times \text{Integral}_1 + C_2 \times \text{Integral}_2 + \dots$$

Integrals have non-zero external momentum $p^2 = m_\mu^2$

multiple propagators

complicated numerator

masses: 0, m_μ , $M_W, \dots, M_{\text{SUSY}}$

⇒ no analytic expressions available

3. Expand integrals in $m_\mu \ll M_{\text{weak}}, m_{\tilde{f}}$

Leading term in a_μ : $\sim \frac{m_\mu^2}{M_{W,\text{SUSY}}^2} \approx 10^{-6}$

$\Rightarrow \frac{m_\mu}{M_{W,\text{SUSY}}}$ is a good expansion parameter

\Rightarrow perform Taylor expansion or “Large mass expansion” of the integrals

\Rightarrow all integrals are reduced to vacuum integrals: T_{134}, A_0, B_0

- Further simplification of coefficients
- Insertion of analytical expressions for T_{134}, A_0, B_0

\Rightarrow Analytical result for $a = (g - 2)_\mu$

\Rightarrow numerical evaluation possible

(implementation in *FeynHiggs2.1*)

Performed checks of our result:

- Cancellation of UV divergences
- Cancellation of terms $\sim m_\mu^{-4}, m_\mu^{-2}, m_\mu^0$
- Cancellation of field renormalization constants
- Reevaluation of SM 2L diagrams from [Czarnecki, Krause, Marciano '98]
⇒ perfect analytical agreement (after going to the appropriate limit)
- Going to the limit of [A. Arhrib, S. Baek '01] ⇒ agreement
(i.e. [C. Chen, C. Geng '01] is too large by a factor of 4)

4. MSSM two-loop results

Our two questions:

Q1: How large are the complete contributions?

Q2: What happens if experimental constraints are taken into account?

⇒ Scan over the MSSM parameter space

$$-3 \text{ TeV} \leq \mu \leq 3 \text{ TeV}$$

$$-3 \text{ TeV} \leq A_{t,b} \leq 3 \text{ TeV}$$

$$150 \text{ GeV} \leq M_A \leq 1 \text{ TeV}$$

$$0 \leq M_{\text{SUSY}} \leq 1 \text{ TeV}$$

$$\tan \beta = 50$$

$$M_{\text{SUSY}} = M_Q = M_L = M_U = M_D = M_E \text{ (later relaxed . . .)}$$

$$A_\tau = A_b$$

Experimental constraints:

Quantity	M_h	$\Delta\rho^{\text{SUSY}}$	$\text{BR}(B_s \rightarrow \mu^+\mu^-)$	$\Delta_{B \rightarrow X_s\gamma}$
strong bound	$> 111.4 \text{ GeV}$	$< 3 \times 10^{-3}$	$< 0.97 \times 10^{-6}$	$< 1.0 \times 10^{-4}$
weak bound	$> 106.4 \text{ GeV}$	$< 4 \times 10^{-3}$	$< 1.2 \times 10^{-6}$	$< 1.5 \times 10^{-4}$

M_h : strong: exp. bound - 3 GeV theory uncertainty

weak: effect of $\delta m_t^{\text{exp}} = +5 \text{ GeV}$

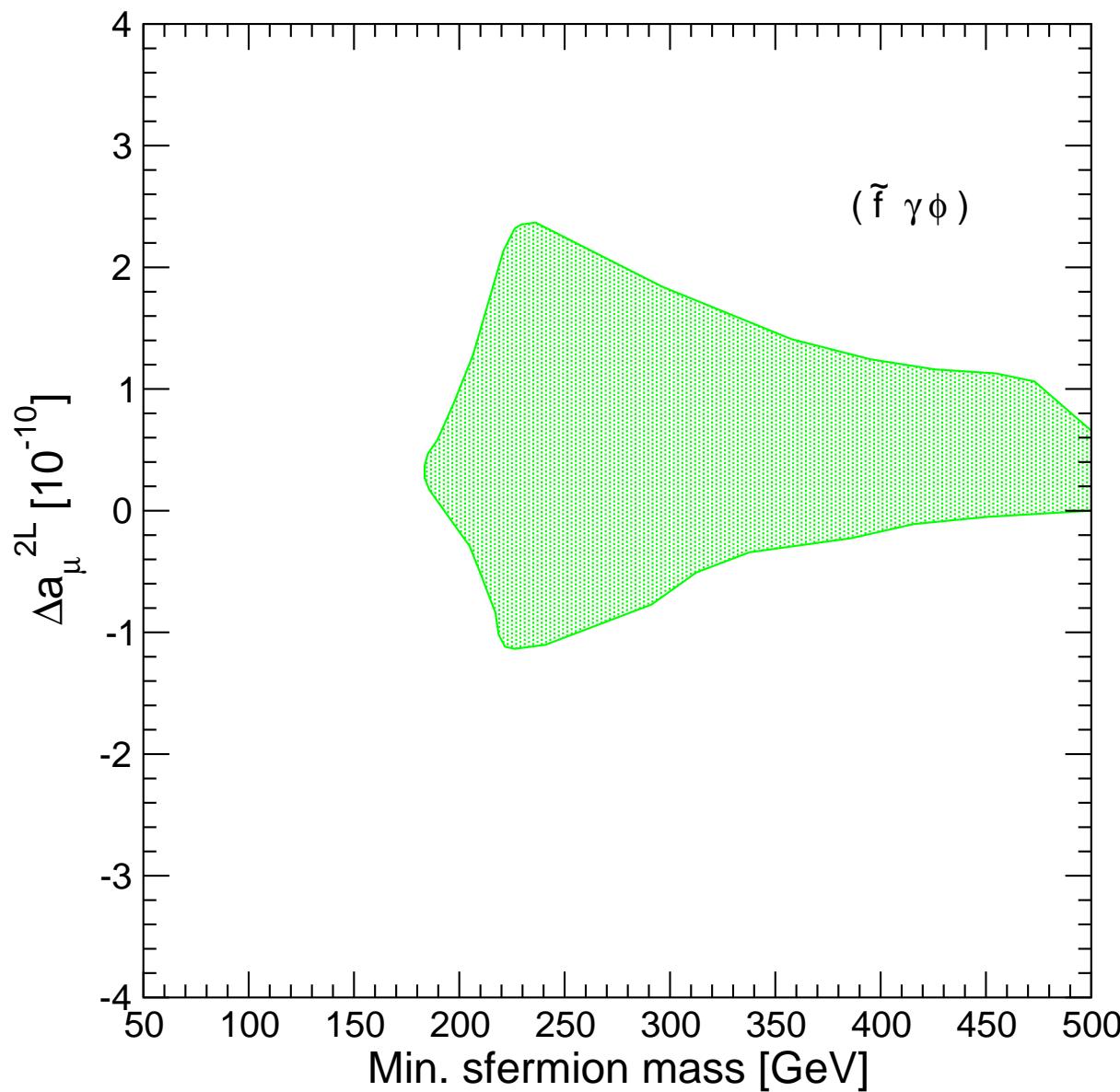
$\Delta\rho^{\text{SUSY}}$: strong: 2σ , weak: 3σ

$\text{BR}(B_s \rightarrow \mu^+\mu^-)$: strong: 90% CL, weak: 95% CL

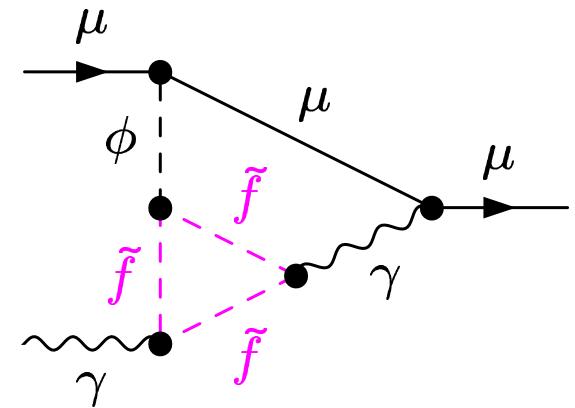
$\Delta_{B \rightarrow X_s\gamma} = |\text{BR}(B \rightarrow X_s\gamma) - 3.34 \times 10^{-4}|$

strong: 90% CL, weak: 95% CL

Numerical results with strong bounds (I)

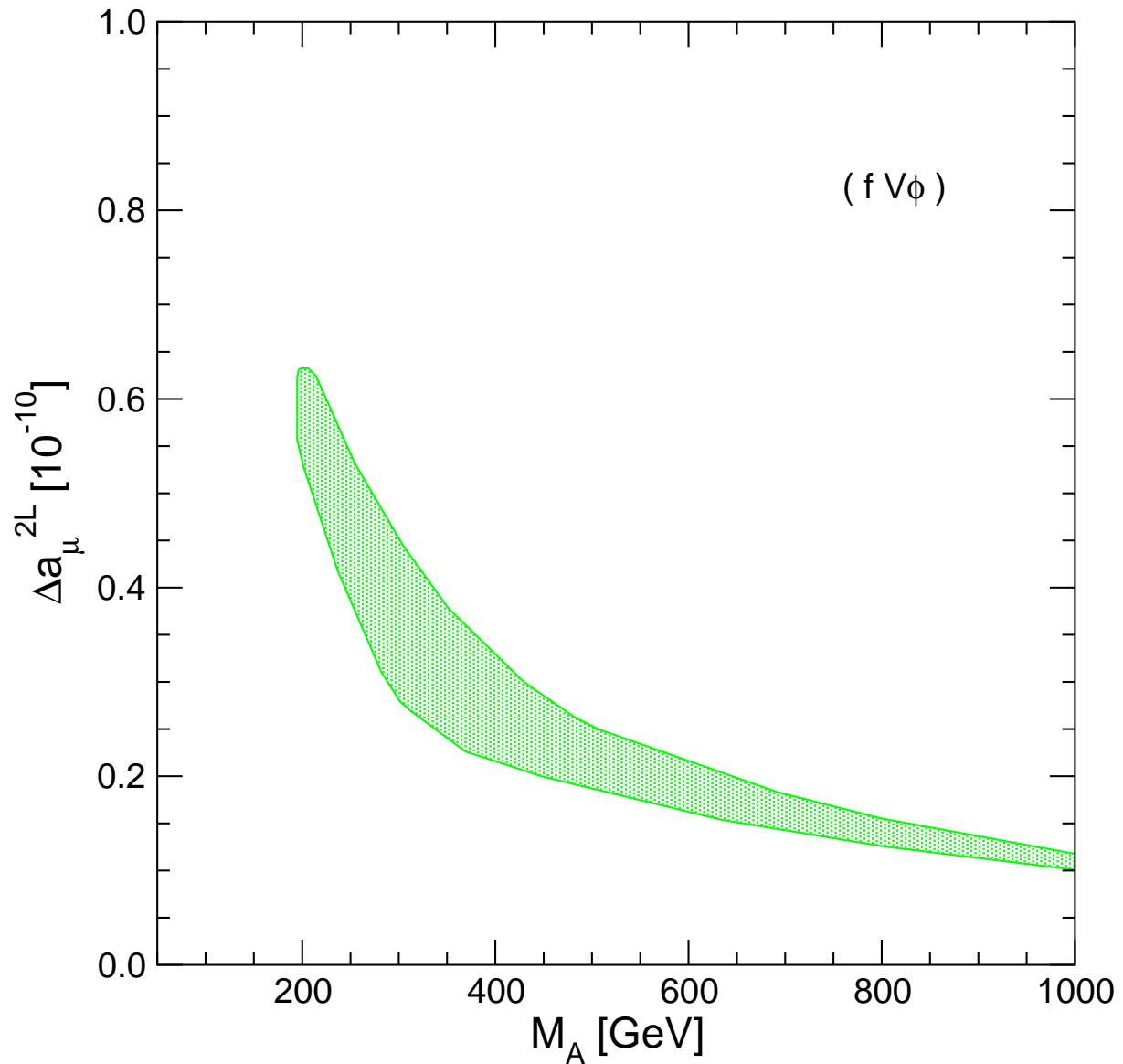


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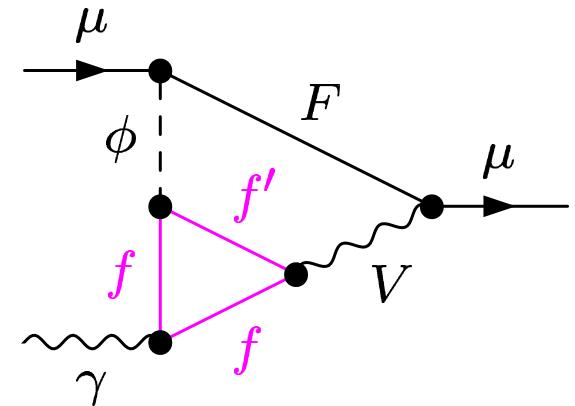


most important
depending on $m_{\tilde{f}}$, μ , A_f ,
 $\tan \beta$
significant fraction of
current experimental error
(Min. sferm. mass =
 $\min\{m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_{\tilde{b}_1}, m_{\tilde{b}_2}\}$)

Numerical results with strong bounds (II)



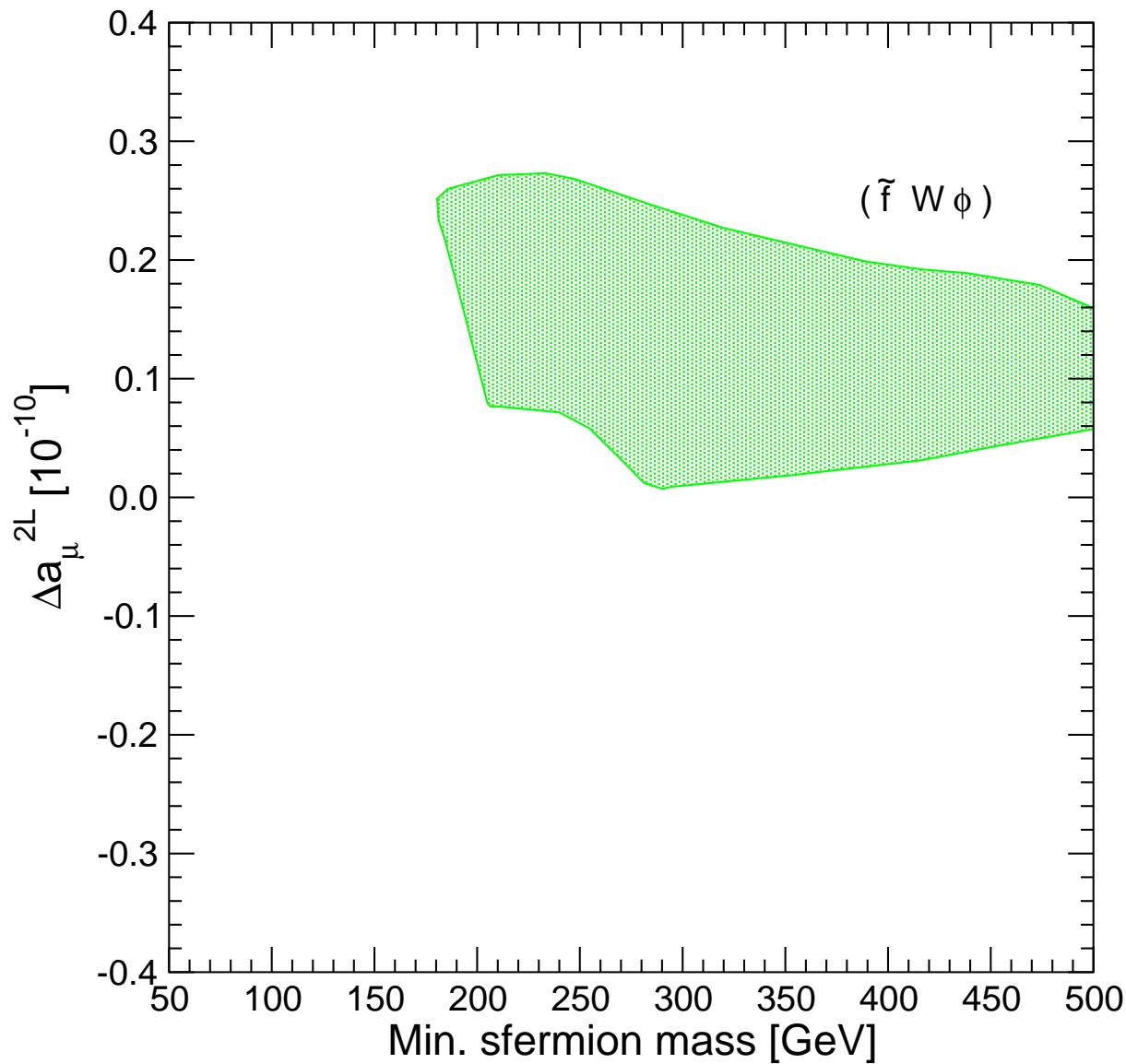
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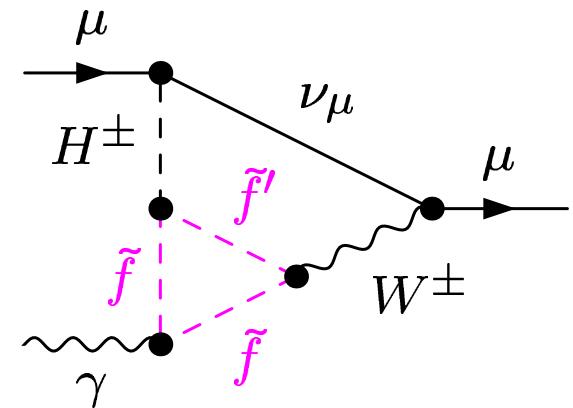
non-negligible
depending on $M_A \dots$

(shown here:
MSSM - SM contribution)

Numerical results with strong bounds (III)



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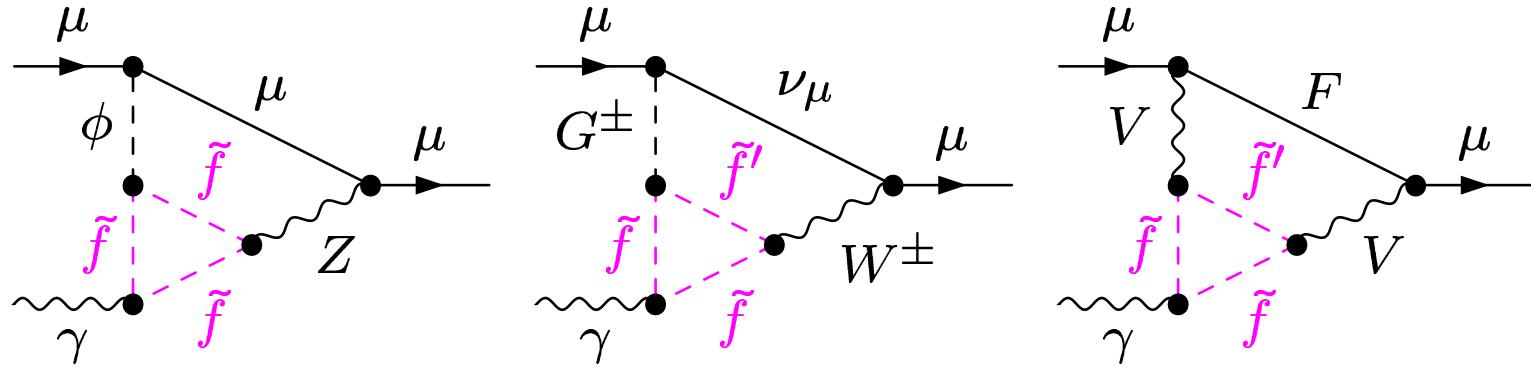


small
depending on $m_{\tilde{f}}$, μ , A_f ,
 $\tan \beta$

(Min. sferm. mass =
 $\min\{m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_{\tilde{b}_1}, m_{\tilde{b}_2}\}$)

Numerical results with strong bounds (IV)

Other contributions: $\lesssim 0.1 \times 10^{-10}$



$((\tilde{f}VV))$ also includes Δr contribution from reparameterization of one-loop result)

⇒ concentrate on $(\tilde{f}\gamma\phi)$, $(\tilde{f}W\phi)$ for further investigations

$\Delta a_\mu^{2L} \lesssim 3 \times 10^{-10}$ for strong constraints

Q2: What are the effects of the experimental constraints?

⇒ apply weak constraints ⇒ apply strong constraints

A2: Effects of weak experimental constraints:

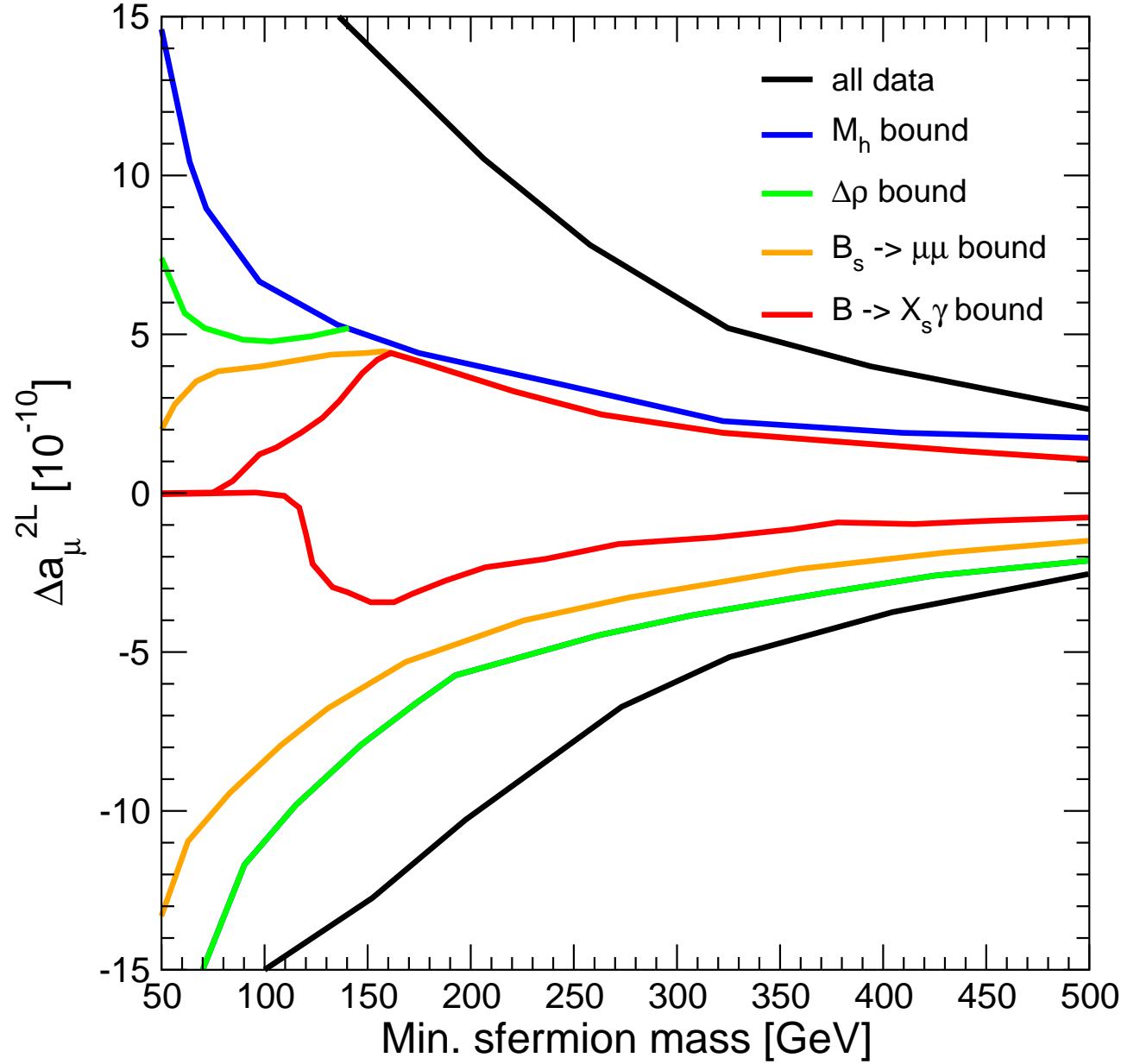
- take the whole data sample
- apply more and more **weak** experimental constraints

→ T

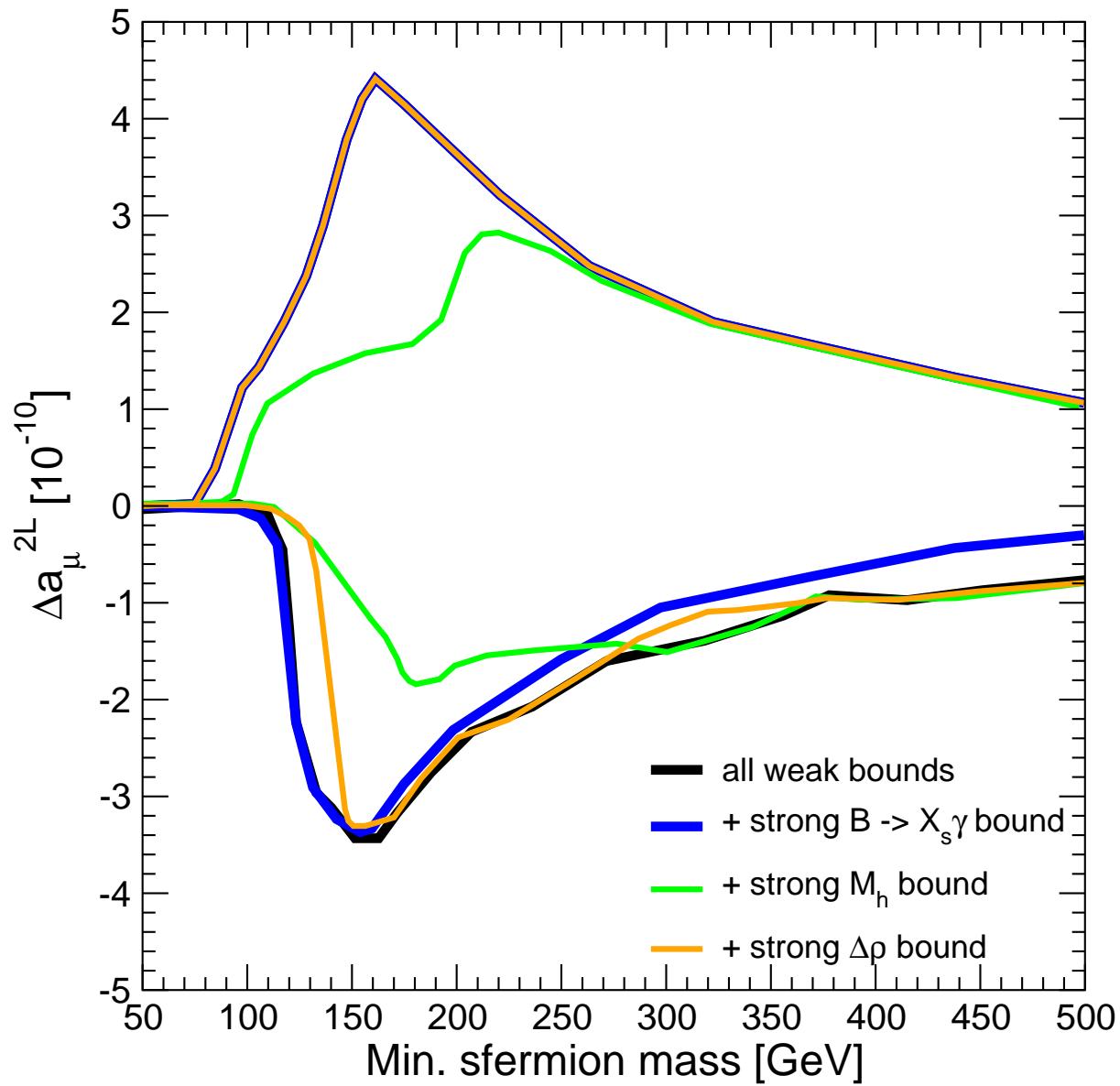
Observations:

- no constraints ⇒ corrections up to 20×10^{-10} possible
- M_h bound very strong, $|\Delta a_\mu^{2L}| \lesssim 5 \times 10^{-10}$ for $m_{\tilde{f}} \gtrsim 150$ GeV
- $\text{BR}(B \rightarrow X_s \gamma)$ bound cuts away everything for $m_{\tilde{f}} \lesssim 150$ GeV
- weak bounds ⇒ $|\Delta a_\mu^{2L}| \lesssim 5 \times 10^{-10}$

Application of weak constraints:



A2': Effects of strong experimental constraints:



Most effective: M_h bound

⇒ reduction to

$$|\Delta a_\mu^{2L}| \lesssim 3 \times 10^{-10}$$

(up to $1/2\sigma$ of current experimental precision)

A2": How to "avoid" the experimental constraints:

Main reason for “small” results: experimental bounds constrain $|\mu| \lesssim 1$ TeV

However:

more freedom for μ with non-universal soft SUSY-breaking parameters

Example (investigated here):

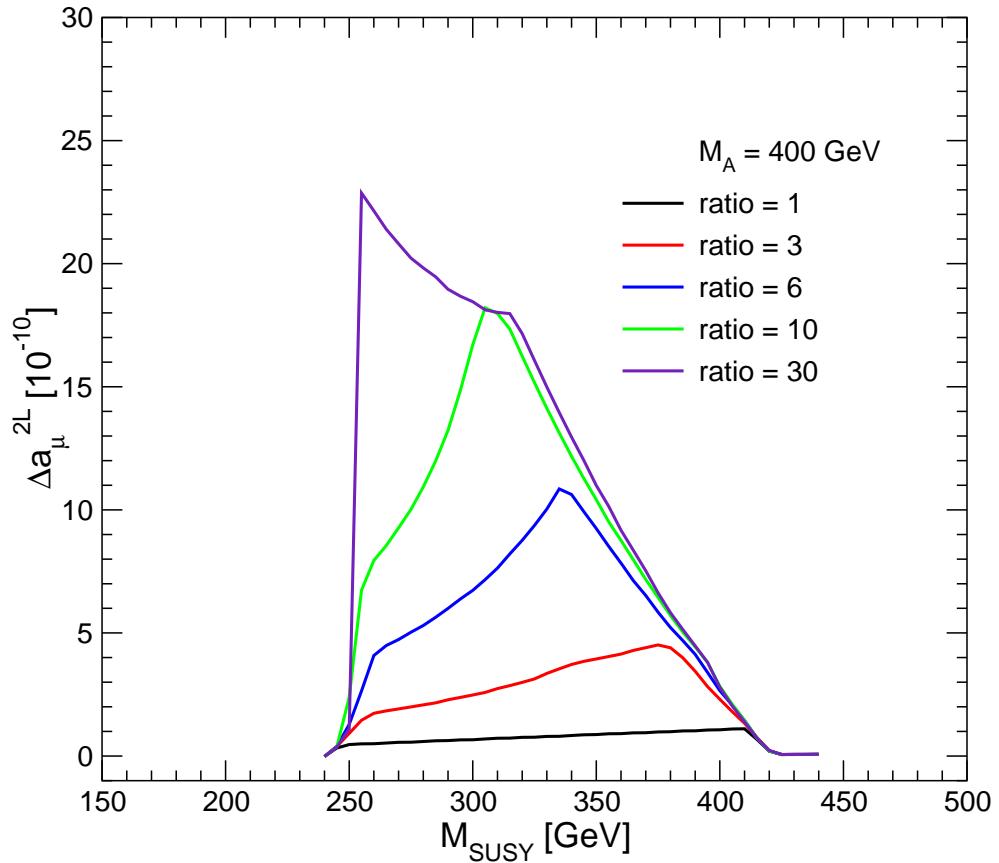
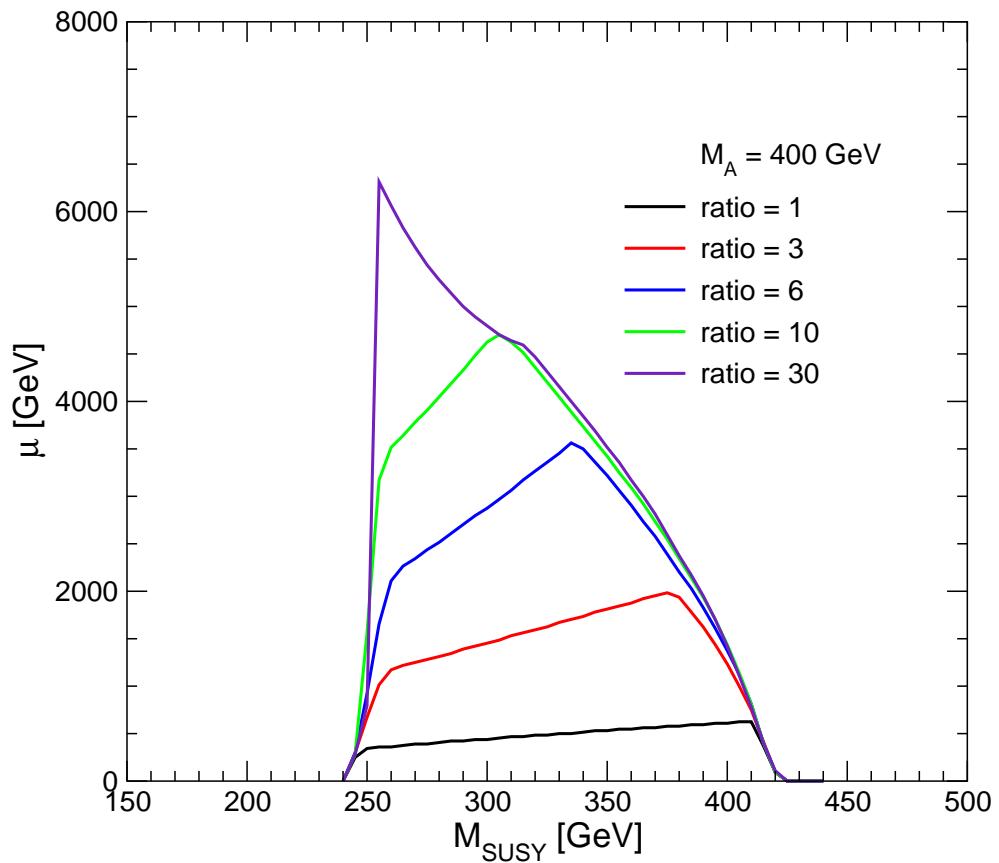
$$M_{\text{SUSY}} = M_Q = M_U = M_L \neq M_D = M_E$$

(disconnects \tilde{t} and \tilde{b} sector)

To obtain “extreme” results:

$$M_A = 400 \text{ GeV}, m_{\tilde{t}_1} = 150 \text{ GeV}, \text{ratio} := M_D/M_U \neq 1$$

Results for relaxed universality condition:



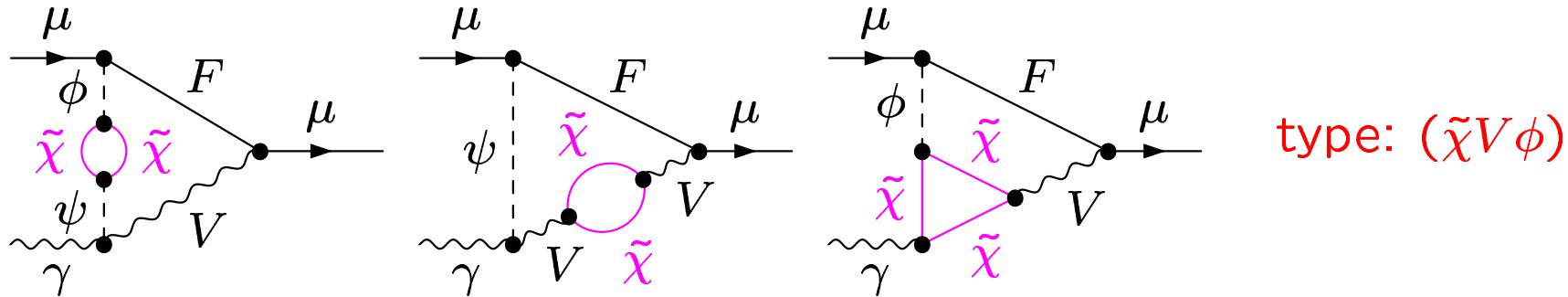
$M_D/M_U \gg 1 \Rightarrow$ very large μ possible

$\Rightarrow \Delta a_\mu^{2L} > 20 \times 10^{-10}$ possible

(this shows how “difficult” it is to obtain large corrections)

Very recent results

All diagrams with a **closed chargino/neutralino loop**:



type: $(\tilde{\chi} V \phi)$

Approximation formula:

$$\Delta a_{\mu}^{2L\tilde{\chi}} \approx 0.27 \times 10^{-10} \times \left(\frac{100 \text{ GeV}}{M_{\text{SUSY}}} \right)^2 \times \tan \beta \times \text{sgn}(\mu)$$

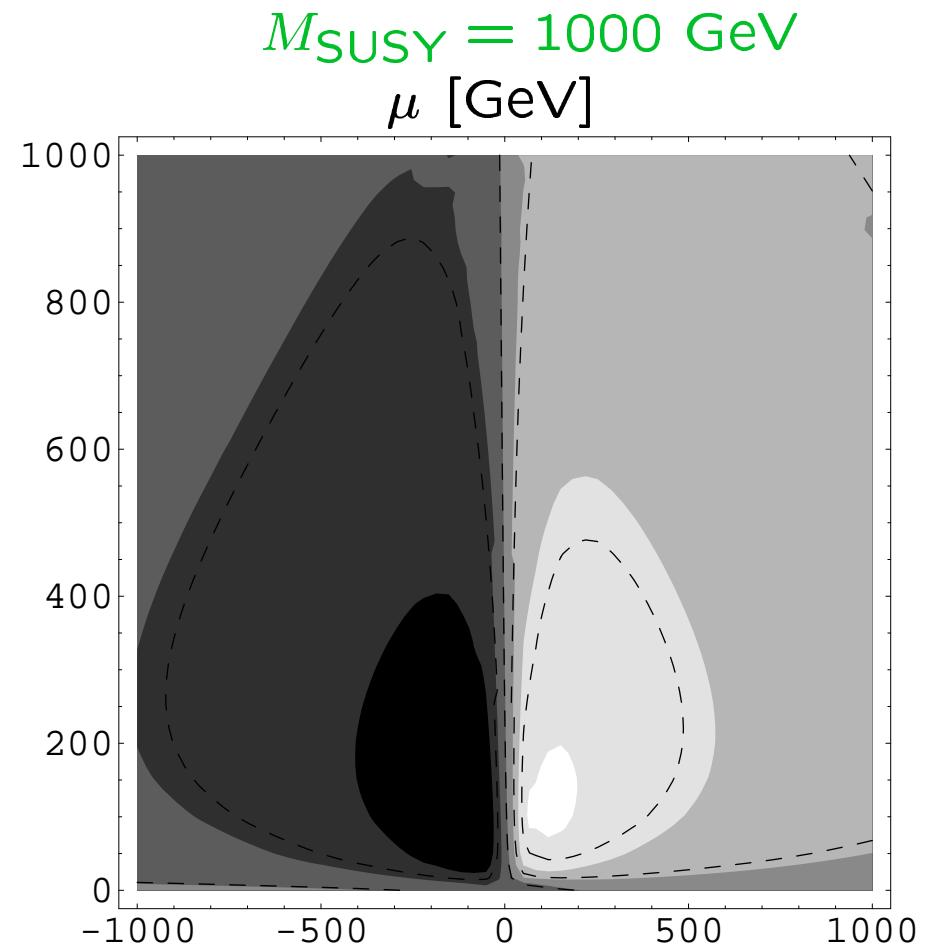
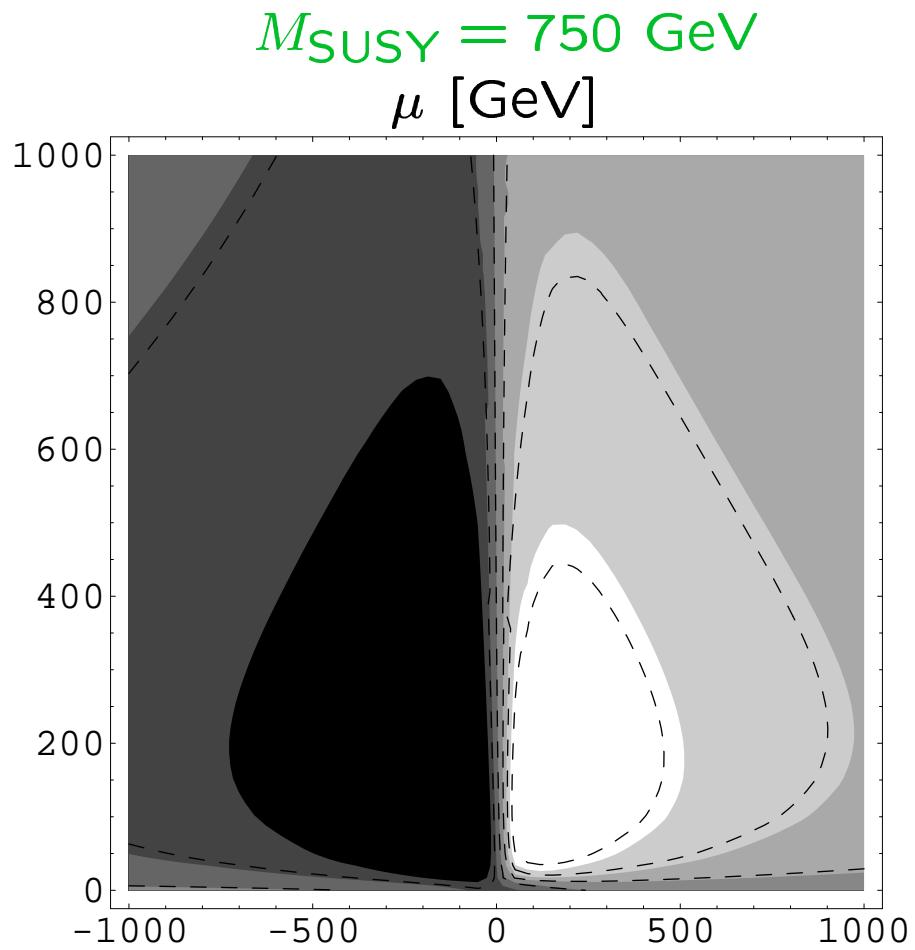
Same particles as at one-loop \Rightarrow add one- and two-loop corrections

Parameter:

$M_{\text{SUSY}}(\text{3rd gen.}) = 1000 \text{ GeV}$, $M_A = 200 \text{ GeV}$, $\tan \beta = 50$

$\rightarrow T$

μ - M_2 plane: indicated: 1, 2, 3- σ areas:



dashed: one-loop only

colored areas: one+two-loop \Rightarrow non-negligible effect

5. Conclusions

- Precision observables can give valuable information about the “true” Lagrangian
- new experimental result for a_μ :
 $a_\mu^{\text{exp}} - a_\mu^{\text{theo}} \approx (29 \pm 10) \times 10^{-10}$: $2.5 - 3.4\sigma$ (e^+e^- data, no MV)
- SUSY could easily explain “discrepancy”
 a_μ can provide bounds on SUSY parameters
- SUSY enhancement factors: $m_\mu \tan \beta \propto \mu m_t$, $A_b m_b \tan \beta$
- Evaluation of two-loop contributions with closed f/\tilde{f} loop
- Expansion \Rightarrow analytic result in vacuum integrals
- strong exp. bounds $\Rightarrow |\Delta a_\mu^{2L}| \lesssim 3 \times 10^{-10}$ ($\sim 1/2\sigma$ of exp. error)
- weak exp. bounds $\Rightarrow |\Delta a_\mu^{2L}| \lesssim 5 \times 10^{-10}$
- breaking of universality of SUSY-breaking terms $\Rightarrow \Delta a_\mu^{2L} \lesssim 20 \times 10^{-10}$

Outlook: evaluation of further two-loop corrections